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13. ABSTRACT (Maximum 200 words)

Research on this contract was directed towards areas of mathematics and numerical computation which have applications to image/signal processing. The research can be broadly classified into the following areas: (1) sparse representations of functions and data, (2) adaptivity for generating sparse representations, (3) learning theory, (4) compression of digital elevation maps, and (5) wireless communication.

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MATHEMATICAL ANALYSIS FOR DATA AND IMAGE PROCESSING

ARO Contract DAAD 19-02-1-00028

Principal Investigator: Ronald A. DeVore
Co–Principal Investigator: Robert C. Sharpley

FINAL PROGRESS REPORT
Scientific Progress and Accomplishments

01 March 2002 – 31 December 2004

STATEMENT OF THE PROBLEM STUDIED

Research on this contract was directed towards areas of mathematics and numerical computation which have applications to image/signal processing. The research can be broadly classified into the following areas: (1) image processing, (2) sparse representation and encoding for digital elevation maps, (3) adaptivity, (4) learning theory, and (5) wireless communication.

SUMMARY OF THE MOST IMPORTANT RESULTS

1. Image Processing

Most tasks in image processing rest on finding a sparse representation of the image in terms of elementary building blocks. Wavelet bases and some of their descendants such as curvelets and ridgelets have proven to be simple, computationally fast, and very effective. For example they are an integral part of the latest JPEG and MPEG standards. The question arises in what sense are wavelet bases best. A program to answer this and other questions was initiated by Donoho [D]. However, Donoho's theory was limited to representations which take place in a Hilbert space. In [DPT], we extend the Donoho theory in a very nontrivial way to sparse representations in general Banach spaces. We show that when a class of functions K has an unconditional basis B^* in the metric of K as well as the metric in which the approximation is to take place, then n-term approximation using the elements of B^* is at least as good as n-term approximation from any other basis

B. Several generalizations of this result are also proven. We also bring out the connections between the best basis problem and encoding of the elements of the set K; that is the problem of representing the elements of K economically by bitstreams. The theory we develop has obvious implications to compression of images and data.

We have also studied the stability of greedy pursuit in the BV norm. This norm is commonly used in image processing tasks such as denoising. We show in [BDKPW0] that greedy projections are stable in BV thus solving a problem of Yves Meyer.

2. Digital Elevation Maps

One of the focal applications of this research is the compression of Digital Elevation Maps (DEMs). DEMs are usually rendered as 3-D surfaces and image processing techniques are not appropriate for processing these maps. We have stressed the importance of developing data compression in the framework of new metrics (such as the Hausdorff metric) which incorporate the geometry in DEMs and are also more pointed to their intended applications.

Our research has been directed at two fronts. The first is to determine the Kolmogorov entropy in the Hausdorff metric for various model classes for DEMs. This has led to many interesting results [CDDD] for classes such as BV or piecewise smooth functions. To complete this direction, we want to incorporate more geometry into the model classes since we feel this captures the spirit of DEMs.

The second front of our research in DEM compression is directed at the development of algorithms and encoders for DEMs. Some of the desired features of the algorithms under development are: (a) high compression, (b) robust error handling, (c) progressive transmission of the data, (d) quick rendering, and (e) burning in (tunnelling) and line of sight display. Since almost all graphic hardware uses triangular polygonal patches as building blocks for object description, we focus our attention to algorithms utilizing meshes of polygonal elements. We have investigated several types of algorithms and encoders:

- Nonlinear approximation algorithms based on adaptive multiresolution analysis;
- Greedy (insertion or removal) algorithms for mesh construction which utilize Delaunay triangulation;
- Progressive encoding based on level sets.

The first algorithms includes: (a) initial coarse adaptive triangulation which allows a low resolution good approximation, (b) wavelet decomposition of the function for achiev-

ing sparse representation of the function (surface), (c) conversion to hierarchical B-spline representation and application of the nonlinear uniform approximation scheme from [DJL] and [DPY], and (d) compression and progressive transmission of the data using the hierarchical representation.

The greedy removal algorithm is a recursive procedure with the following basic elements: (a) determination and upgrading of the significance table of the grid points, (b) removal one by one of the least significant points, and (c) mesh updating after each removal with Delaunay triangulation algorithm. The greedy insertion algorithm utilizes the same elements but in a reverse order. We pay special attention to the data structure that enables us to compress and transmit the data progressively.

The level set method seeks first to give a progressive description of the surface in terms of level curves and Morse trees. We prioritize the level curves and ridge curves and then encode each of them in a progressive manner. The remainder of the surface is then extrapolated from this information by blending or interpolation.

The level curves are compressed using a multiscale decomposition as described in [BDDD]. This paper also proves various theorems which prove the efficiency of this method of representing and encoding curves.

A major question that we are studying in detail is to understand which surfaces can be compressed well using level set methods. In this direction, we have introduced new anisotropic spaces of functions in [BDKPW] and shown that surfaces which are graphs of these functions can be compressed well with level set methods. These new anisotropic spaces are completely different from the anisotropic spaces usually studied in harmonic analysis and PDEs. For example functions with large gradients are in a certain sense nice functions with respect to this family of spaces. The correct description of these spaces when measuring higher order smoothness is still to be completely worked out. We believe that these spaces will play an important role in analysis, not only for surface compression but also for the study of nonlinear evolution equations.

3. Adaptivity

One way of generating sparse data dependent approximations is to use adaptive refinement. Starting with a coarse triangulation of a domain, an adaptive method generates new triangulations with finer and finer resolution by refining where the data changes a lot (regions of nonsmoothness or singularities) and does not refine where the data does not change much (regions of smoothness). The partitions generated by adaptive methods can be described by trees. Our goal in this research has been to find adaptive partitions that are nearly best and to find them with minimal computation time. In [BD], we describe an

adaptive tree approximation algorithm which finds near best trees in linear time. That is, the computational time is proportional to the number of nodes in the tree. Of course, this cannot be improved. This algorithm has already found several impressive applications. One of these is for adaptive methods for PDEs. We give in [BDD] the first known adaptive algorithm for solving elliptic problems that is proven to have optimal approximation rates. In [BDDP], we describe the class of functions which can be approximated with a given rate $O(n^{-s})$ by adaptive approximation using n triangles in the adaptively generated partition. The characterization is in terms of membership in Besov spaces. This theorem is important in both numerical PDEs and in data processing since it tells us what rates of approximation/compression we can expect when using adaptive techniques.

4. Learning Theory

A typical application of surface processing is to generate a faithful representation of noisy point cloud data associated to a given surface. This can be viewed as a regression problem in learning theory where the unknown underlying probability distribution corresponds to the noisy data. The noise arises from sensor noise, sensor jitter, error in global positioning, misclassification of points on the surface, etc. We have developed in [DKPT] a general theory which describes when learning algorithms are optimal and gives the theoretical framework for creating optimal algorithms. In [BCDDT] we have developed an adaptive algorithm (an alternative to model selection) which is shown to be optimal (in a certain sense) for learning the regression function from a given data set. This technology has been applied to learning surfaces generated in real time in the autonomous navigation of Micro Air Vehicles (MAVs) (see [KNPDBDS]).

5. Wireless Communication

Wireless communication will play an increasing role in future conflicts. There are two critical elements: compression (in order to communicate over narrow bandwidths), and bit protection (to be able to recover the signal even when bits are lost). Our view is that these two elements should not be treated separately but should rather be done in tandem. We have used multiscale methods and redundant systems (frames) for these tasks. For example, in [DPS] we have shown how to utilize multiscale methods to progressively encode flow fields in streaming video. We also have good first results for incorporating bit protection into a redundant frame [P].

PUBLICATIONS AND TECHNICAL REPORTS

(a) Papers published in peer-reviewed journals

P. Binev, W. Dahmen, R. DeVore, and P. Petrushev, “Approximation Classes for Adaptive Methods,” Serdica Math. J. **28** (2002), pp. 391–416.

Albert Cohen, Wolfgang Dahmen, and Ronald DeVore “Sparse Evaluation of Compositions of Functions Using Multiscale Expansions,” SIAM J. Math. Anal., Vol. 35 (2003), pp. 279–303.

Albert Cohen, Wolfgang Dahmen, and Ronald DeVore, “Adaptive Wavelet Schemes for Nonlinear Variational Problems,” SIAM J. Numer. Anal., Vol. 41, No. 5 (2003), pp. 1785–1823.

P. Binev, W. Dahmen, and R. DeVore, “Adaptive Finite Element Methods with convergence rates,” Numerische Mathematik, **97** (2004), pp. 193–217.

P. Binev and R. DeVore, “Fast computation in tree approximation,” Numerische Mathematik **97** (2004), 219–268.

(b) Papers published in non-peer-reviewed journals or in peer-reviewed conference proceedings

R. DeVore, A. Petukhov, and R. C. Sharpley, “Motion Estimation with the Redundant Wavelet Transform,” Proc. of the 3rd Workshop on Digital and Computational Video, Clearwater Beach, Nov. 14-15, 2002.

P. Binev, W. Dahmen, R. DeVore, and N. Dyn, *Adaptive Approximation of Curves*, in APPROXIMATION THEORY: A volume dedicated to Borislav Bojanov, (D. K. Dimitrov, G. Nikolov, and R. Uluchev, Eds.) Marin Drinov Academic Publishing House, Sofia (2004), pp. 43–57. (IMI Preprint Series 2004:19, University of South Carolina).

A. Kurdila, M. Nechyba, R. Prazenica, W. Dahmen, P. Binev, R. DeVore, and R. Sharpley, “Vision-Based Control of Micro–Air–Vehicles: Progress and Problems in Estimation,” 43rd IEEE Conference on Decision and Control, Dec. 14-17, 2004, Atlantis, Paradise Island, Bahamas, pp. 1635–1642.

(c) Papers presented at meetings, but not published in conference proceedings

none

(d) Manuscripts submitted, but not published

P. Bechler, R. DeVore, A. Kamont, G. Petrova, P. Wojtaszczyk, "Greedy wavelet projections are bounded on BV," Trans. Amer. Math. Soc., to appear.

P. Bechler, R. DeVore, A. Kamont, G. Petrova, and P. Wojtaszczyk, "Anisotropy and the approximation and encoding of surfaces," in preparation.

P. Binev, A. Cohen, W. Dahmen, R. DeVore, and V. Temlyakov, "Universal Algorithms for Learning Theory, Part I: Piecewise Constant Functions," IMI Preprint Series 2004:20, University of South Carolina.

R. DeVore, G. Kerkyacharian, D. Picard, and V. Temlyakov, "On Mathematical Methods for Supervised Learning," IMI Preprint Series 2004:10, University of South Carolina.

(e) Technical reports submitted to ARO

YEAR 1 Interim Progress Report DAAD19-02-1-00028 (submitted March 27, 2003)

YEAR 2 Interim Progress Report DAAD19-02-1-00028 (submitted March 30, 2004)

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REPORT OF INVENTIONS

None to Report; See DD Form 882, attached.

TECHNOLOGY TRANSFER

The technology transfer of encoding for surfaces by level curves and registration of surfaces from sensors to the DOD laboratories and contractors has been initiated through the Phase I STTR program with Schafer Corporation, a mid-sized defense contractor. An application is in for Phase II with Schafer Corporation to develop commercial code for the same projects of encoding and registration.

In addition to Defense and Homeland security applications, there is potential for commercial interest for significant improvements in Geographical Information Systems, particularly for improved database queries, navigation, and storage of large scale data.

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[BD] P. Binev and R. DeVore, *Fast computation in tree approximation*, Numerische Mathematik **97** (2004), 219–268.

[CDDD] A. Cohen, W. Dahmen, I. Daubechies, R. DeVore, *The Kolmogorov entropy of model classes of surfaces in the Hausdorff metric*, in preparation.

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[KNPDBDS] A. Kurdila, M. Nechyba, R. Prazenica, W. Dahmen, P. Binev, R. DeVore, and R. Sharpley, *Vision-Based Control of MicroAirVehicles: Progress and Problems In Estimation*, 43rd IEEE Conference on Decision and Control (2004), 1635–1642.

[P] A. Petukhov, *Wavelet Frames and Error Correcting Codes*, in preparation.